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Measuring the flight of an arrow using the Acoustic Doppler Shift

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Abstract

Measuring the velocity of an arrow in flight is a task of interest to archers, but can be difficult without specialized equipment. We show how accurate velocity measurements can be made by recording the sound made by the arrow as it travels over one or more microphones. Useful recordings can be made with a data acquisition system or with the sound recorder built into most PC operating systems. Estimates of the drag coefficient of different arrows can be made using several microphones to record each shot.

Keywords: Archery; Acoustics; Doppler Shift

1. Introduction

The velocity of an arrow is of primary concern to any archer. One of the most common reasons for poor accuracy is inconsistent velocities between shots. A well-trained archer is able to achieve nearly the same 'muzzle' velocity over a large number of shots. It is not surprising that serious archers often buy devices called chronographs to measure arrow velocity. A typical model times the passing of the arrow's shadow over two photo-detectors whose spacing is precisely known.

An alternate approach is to measure the Doppler shift [1] of the arrow as it passes over one or more microphones. While arrows naturally make noise in flight, that noise is broadband—possibly due to turbulent flow over the feathers at the rear of the arrow. For the Doppler shift measurement to be useful, the arrow should make a single frequency in flight. Fortunately, there are tips available that make a high-pitched whistle in flight. Tips of this type were originally developed to act as signals during battle [2]. They are available now as items of historical interest.

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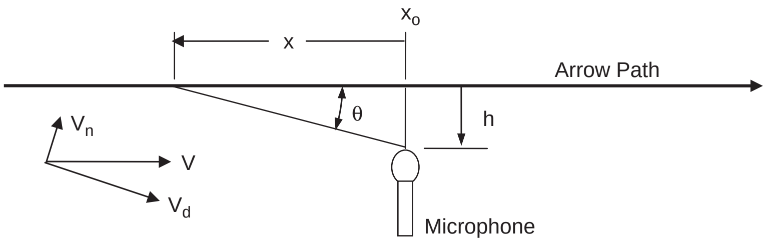

Fig. 1. Geometry of arrow path.

2. Time–frequency relationship

We assume that the arrow's path is approximately straight in the near vicinity of the microphone and can be described as shown in Fig. 1.

The component of velocity toward the microphone, V_d, is the one that generates a Doppler shift. The normal component, V_n, does not and can be ignored for this analysis. Note that $\theta = 90^\circ$ as the arrow passes directly over the microphone. At this instant, $V_d = 0$ so there is no Doppler shift.

Finally, the Doppler-shifted frequency is

$$f = f_s \frac{c}{c - V_d}, \tag{1}$$

where f_s is the source frequency—the frequency of the source at rest and c is the speed of sound. The resulting expression for the shifted frequency as a function of time is

$$f = f_s \frac{c}{c - V \cos[\tan^{-1}(h/(Vt + x_0))]}. \tag{2}$$

3. Measuring arrow velocity

We conducted tests so that Eq. (2) could be correlated with experimental data. We fitted identical tips to two different arrows. Both had nominally identical shafts, but one had standard parabolic feathers and one had untrimmed feathers. The untrimmed feathers create high drag for shots in which the arrow is likely to miss and the shooter does not wish to search too far. The most common example is hunting birds.

Our test setup used three microphones placed along the path from the shooter to the target. We conducted the tests inside a closed room, so the distance from the shooter to the target was limited by the length of the diagonal of the room—approximately 13 m.

Fig. 2 shows a Choi–Williams distribution [3,4] of a whistle-tipped arrow passing over the microphone nearest the shooter. The measured frequency asymptotically approaches 4635 Hz near the left of the figure—when the arrow is far from the microphone. Conversely it asymptotically approaches 3310 Hz as moves far past the microphone.

While the arrow is far from the microphone, $V_d \approx V$. Eq. (2) can be modified to

$$c\left(1 - \frac{f_s}{f}\right) = V_d \approx V \quad \theta \ll 1. \tag{3}$$

If f_1 is the asymptotic frequency as the arrow approaches the microphone and f_2 is the asymptotic frequency as it recedes from it, then Eq. (3) results in

$$V = c\left[\frac{f_1 - f_2}{f_1 + f_2}\right]. \tag{4}$$

The conditions in the room during the test were very close to the assumed standard atmosphere in which the speed of sound is 338.3 m/s. Using this value and Eq. (2), the velocity of the arrow is 56.42 m/s.

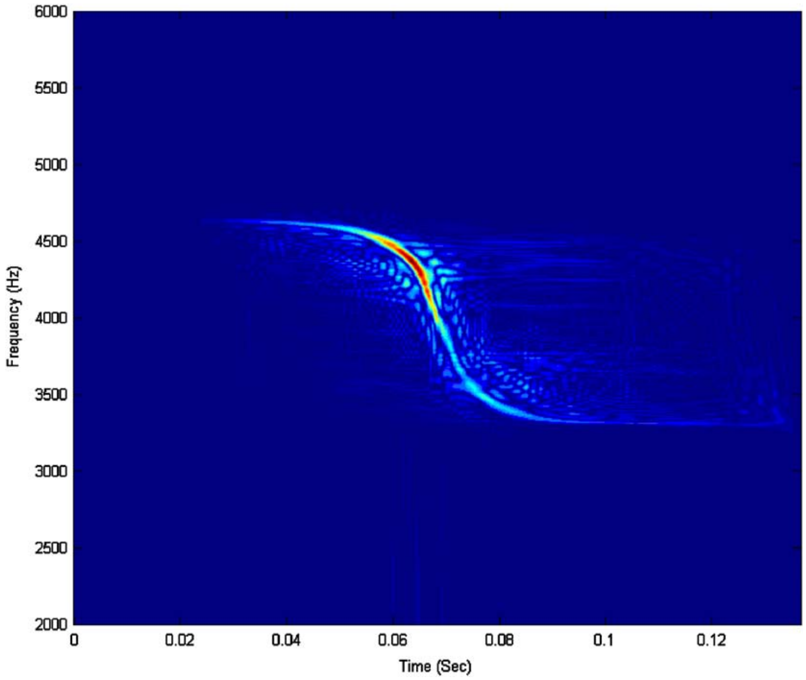

Fig. 2. Choi–Williams distribution of an arrow whistle, $\alpha = 50$.

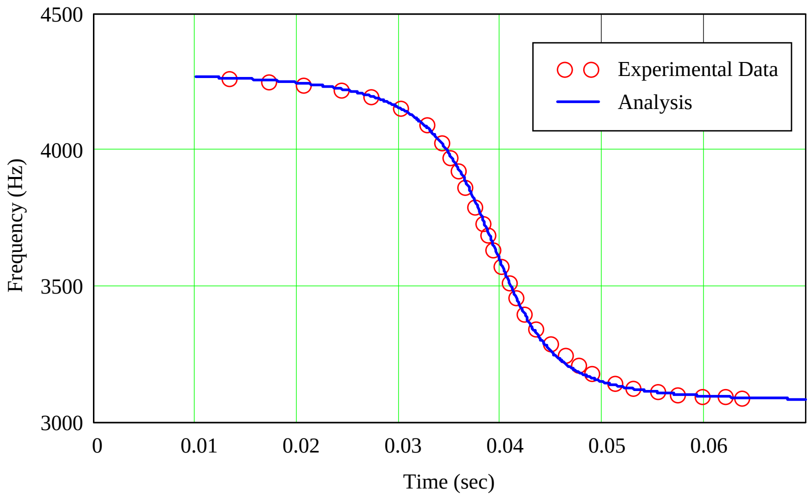

Fig. 3. Experimental data and analytical results.

The experimental data from each of the three microphones was fitted using Eq. (2). Fig. 3 shows a representative result.

Table 1 shows the identified parameters from the two arrows as they were shot over a line of three microphones.

4. Determining drag coefficient

Aerodynamic forces are assumed here to be proportional to the square of the velocity. This assumption is consistent with linear theory and is generally applied to slender bodies with lifting surfaces. Considering only

Table 1
Parameters identified from experimental data

	h (m)	x_0 (m)	f_s (Hz)	v (m/s)	t_0 (s)
Parabolic feathers					
Mic 1	0.467	3.99	3860	56.98	0.0700
Mic 2	0.396	6.95	3850	56.08	0.1239
Mic 3	0.235	8.96	3845	56.08	0.1597
Untrimmed feathers					
Mic 1	0.402	2.23	3635	56.39	0.0395
Mic 2	0.366	5.12	3625	53.95	0.0948
Mic 3	0.168	7.10	3606	53.64	0.1324

the horizontal component of velocity, the equation of motion for the arrow is

$$m_a\dot{v} = -cv^2, \tag{7}$$

where m_a is the mass of the arrow and c is a proportionality constant. For the arrow with parabolic feathers, $c = 3.1 \times 10^{-6}$. For the arrow with untrimmed feathers, $c = 9.4 \times 10^{-6}$.

The distance of the arrow above the microphone, h, is a by-product of identifying the shifted frequency profile as described by Eq. (2). Thus, the sound recordings also contain information that will define the two-dimensional flight profile.

5. Summary

We have presented an accurate means for identifying the flight path of an arrow using only acoustic measurements. Velocity histories and drag coefficients of different types of arrows can be extracted easily from the recorded sound data. While the approach has been demonstrated for arrows, it is general enough to be used for any moving body that makes a Doppler-shifted noise and travels slower than the speed of sound.

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